

Revista de Ciências Agroveterinárias 23 (3): 2024 Universidade do Estado de Santa Catarina

Intensity-Duration-Frequency Relationships of intense rainfall in Lages, SC

Relações Intensidade-Duração-Frequência de chuvas intensas de Lages, SC

Romeu Souza Werner1(0000-0003-4396-5382), Ildegardis Bertol* ² (0000-0003-4396-5382), Fabrício Tondello Barbosa3(0000-0002- 1724-2327)

¹Santa Catarina State University, Lages, SC, Brazil. ²Santa Catarina State University, Lages, SC, Brazil. *Corresponding author: ildegardis.bertol@udesc.br. ³Ponta Grossa State University, Ponta Grossa, Brazil.

Submission: 11/16/2023 | Acceptance: 10/05/2024

ABSTRACT

Knowledge of rainfall intensity is essential for planning hydraulic works. With the aim of filling the lack of this information for conservation planning in rural areas, this work aimed to determine the empirical constants of the heavy rainfall equations using precipitation data, and the adjustments of these equations to the observed precipitation. The rainfall data were obtained in equipment (pluviograph and pluviometer) located to the meteorological at the station of the Center for Agro-Veterinary Sciences, in Lages city, Santa Catarina state, Brazil, from 1989 to 2016 (28 years). To obtain the height and intensity of the rainfall in the return periods of 2, 5, 10, 15, 20, 25, 30, 40 and 50 years, the statistical distribution of Gumbel was applied. Parameters were obtained for rainfall periods of up to 240 minutes (Equation I) and from 360 to 1,440 minutes (Equation II). The frequency of intense rain estimated by the Gumbel method, with adherence tested using the Anderson-Darling test, adjusts to the frequency of rain observed in the Lages city. This provides reliability of the probabilistic distribution of the observed data for certain return times and duration of rainfall. The intensity-duration-frequency (IDF) equation fits rainfall data observed over a 28-year and can be used to predict the maximum average rainfall intensity. The IDF I equation is $i=1,285.65$ Tr^{0.133}(t+14.2)^{-0.87}, applied to rainfall lasting up to 240 minutes, and the IDF II equation is $i=1.923.06$ Tr^{0.188}(t +34.85)^{-0.86}, applied to rainfall lasting between 360 and 1,440 minutes.

KEYWORDS: Gumbel distribution; daily rainfall; return time.

RESUMO

O conhecimento da intensidade das chuvas é essencial para o planejamento de obras hidráulicas. Com o objetivo de suprir a falta destas informações para o planejamento conservacionista em áreas rurais, este trabalho teve como objetivo determinar as constantes empíricas de equações de chuvas intensas utilizando dados de precipitação, e ajustar essas equações às chuvas observadas. Os dados de precipitação foram obtidos em equipamentos (pluviógrafo e pluviômetro) localizados na estação meteorológica do Centro de Ciências Agroveterinárias, na cidade de Lages, Santa Catarina, Brasil, no período de 1989 a 2016 (28 anos). Para obter a altura e intensidade das chuvas nos períodos de retorno de 2, 5, 10, 15, 20, 25, 30, 40 e 50 anos, foi aplicada a distribuição estatística de Gumbel. Foram obtidos parâmetros para períodos de precipitação de até 240 minutos (Equação I) e de 360 a 1.440 minutos (Equação II). A frequência de chuvas intensas estimada pelo método Gumbel, com aderência testada pelo teste de Anderson-Darling, ajusta-se à frequência de chuvas observadas na cidade de Lages. Isto proporciona confiabilidade da distribuição probabilística dos dados observados para determinados tempos de retorno e duração das chuvas. A equação intensidade-duração-frequência (IDF) ajusta-se aos dados de precipitação observados ao longo de 28 anos e pode ser usada para prever a intensidade média máxima de chuva. A equação IDF I é i=1.285,65Tr^{0,133}(t+14,2)^{-0,87}, aplicada a chuvas com duração de até 240 minutos, e a equação IDF II é $i=1.923.06$ Tr $^{0.188}$ (t +34.85)^{-0.86}, aplicada à chuvas com duração entre 360 e 1.440 minutos. **PALAVRAS-CHAVE:** distribuição Gumbel; chuvas diárias; tempo retorno.

Hydraulic work planning aims to control surface runoff in rural and urban areas. For this purpose, heavy rains are necessary in the region. Forecasting heavy rainfall is necessary when considering the spatial and temporal variability of rainfall using the principle of probability (MELLO & SILVA 2013). With information on the maximum average rainfall intensities, it is possible to adequately predict the effect of these rains on runoff. Thus, the hydraulic works required for flood control can be appropriately dimensioned based on the magnitude of the maximum predicted precipitation.

The return time (Tr) of a rainfall event is the inverse of the probability that the rainfall event will be equal to or exceeded at least once in the period. The Tr of a rain event was estimated using the probability of its occurrence after the frequency distribution was adjusted to the extreme data of the historical series. At the same time, in general, the frequency of extreme values in historical rainfall series is in accordance with the "Fisher-Tippet" type I distribution (VILLELA & MATTOS 1975), known as the "Gumbel" distribution.

Heavy rainfall is differentiated in terms of intensity (I), duration (D), and frequency (F) of occurrence and can be represented by intensity-duration-frequency curves (IDF equations). These curves were obtained by analyzing a long series of extreme rainfall data. The lack of long-series rainfall data due to the scarcity of collection points at climatological stations and the short period of records makes it difficult to adjust forecast models. The lack of such data makes it difficult to adequately design hydraulic works (SILVA et al. 1999 and OLIVEIRA et al. 2005). In addition to the need for relatively long historical series, the dataset has regional characteristics, which makes hydraulic planning even more difficult.

IDF curves based on historical rainfall records can be obtained using two main methodologies: The first consists of disaggregating the rainfall of a day into shorter periods using specific coefficients (CETESB 1979). By fitting a distribution of extreme values (Gumbel, GEV, Log-gumbel), the height and maximum intensity of rainfall were estimated for periods of less than 24 h. In the second methodology, rainfall records are used to identify the highest intensities in each period. For this purpose, maximum intensities are used to fit the equations, considering the duration and recurrence time.

The equation adjustment method, developed from rainfall records, offers greater reliability in terms of results, as they are rain sheets and their actual durations for the study site. However, this type of equation fitting is sometimes limited when plotting IDF curves. This limitation was due to the low availability of meteorological stations with rain gages and the difficulty in processing the data contained in the rainfall records due to the large volume of information (MELLO & SILVA 2013).

The maximum average rainfall intensity behaves inversely to the duration of the event (TUCCI 1993). Therefore, when correlating only the intensity values with the duration values, for a given return period, it is found that the greater the intensity, the shorter the duration of the rain, configuring a curve with a hyperbolic shape.

Based on this, we hypothesized that an intensity-duration-frequency equation would fit the heavy rainfall data observed at the meteorological station of the Center for Agro-Veterinary Sciences in Lages, Santa Catarina, Brazil. In this way, empirical constants can be obtained, and the adjusted model can be used to support the planning and dimensioning of hydraulic works at this location.

MATERIAL AND METHODS

To adjust the parameters of the general heavy rainfall equation, rainfall heights recorded over 28 years (1989-2016) were used. Data on rainfall height, duration, and intensity were obtained from a rainfall diagram at the Center for Agro-Veterinary Sciences station in Lages, Santa Catarina, Brazil. Nonparametric tests of randomness, stationarity, and double-mass analysis were performed to identify the adequacy and viability of the data in relation to the probability distribution.

The first non-parametric test applied was the randomness of the data to identify whether the hydrological magnitude was due to natural causes, which was verified using the methodology proposed by MELLO & SILVA (2013). The test is based on hypothesis H0, through which it is inferred that historical rainfall series are made up of random data. The second nonparametric test applied to the rainfall data series was stationarity, with the aim of identifying temporal trends and/or clear cycles within the study period. The third test was double mass analysis, which was used to check the homogeneity of the data. This test made it possible to identify whether the variations were due to anomalies at the rainfall station. The methodology used for this test was developed by the US Geological Survey and was based on the construction of a double cumulative curve. This double curve relates the annual totals of accumulated rainfall from the 28-year historical series with data from the Lages Experimental Station, which is part of the Environmental Resources and Hydrometeorology Information Center of Santa Catarina, Brazil. The nonparametric test results express the requirements that historical series

must meet to obtain reliable adjustments with the applied forecasting models. The tests allowed us to verify the suitability and viability of the data for the study, thus allowing us to obtain the parameters for the two IDF equations.

"Equation I" was adjusted to predict average values of maximum rainfall intensity for durations of up to 240 min (4 h). The second, called "Equation II", was adjusted to predict average values of maximum rainfall intensity for durations between 360 and 1,440 minutes (6 to 24 h). With the maximum daily precipitation values, an annual series of extreme values was constructed using data from the computer program (unpublished) of the Soil Department of the Federal University of Rio Grande do Sul (UFRGS). This program provided the maximum precipitation heights for periods of 5, 10, 15, 20, 25, 30, 40, 50, 60, 90, 120, 150, 180, and 240 min, assuming the limit values established by WILKEN (1978). Annual precipitation data were assessed for adherence to the Gumbel distribution using the Anderson-Darling test (p<0.05). The maximum rainfall intensity values that could be equal or exceeded were calculated for return times of 2, 5, 10, 15, 20, 25, 30, 40, and 50 years.

The frequency distributions of the rainfall data and observed frequencies were obtained using the Kimball method, and the theoretical frequencies were obtained using the Gumbel probability distribution. To obtain the value of the reduced variable of the Gumbel distribution, the CHOW (1959) frequency distribution function was applied. To evaluate the performance of the adjustment of the theoretical frequencies estimated by the Gumbel distribution to the observed frequencies, the performance coefficient (C) proposed by CAMARGO & SENTELHAS (1997) was used. The coefficient of performance (C) is obtained by the ratio between the product of the Willmott index (d) and the Pearson correlation coefficient (R), that is, C = d R $^{-1}$. The values of the parameters **C**, m, t0, and n of the IDF equation, obtained through the linearization of the IDF curves when applying the anamorphosis process, were adjusted using Excel software by applying the "Solver" tool. The software minimized the mean square error by iteratively adjusting all parameters of the definitive IDF equation.

Equation II was obtained for rainfall duration times (td) greater than 360 and up to 1,440 minutes using the daily rainfall disaggregation method proposed by DAEE-CETESB (1986). Using the maximum daily rainfall values, values were calculated for shorter durations of 360, 480, 600, 720, and 1,440 min using the corresponding coefficient for each rainfall duration. The maximum rainfall intensity for each season was obtained by dividing the calculated rainfall height by the corresponding duration, expressed in mm h⁻¹. With the values obtained by disaggregating the rainfall of a day, sufficient points were obtained to construct the IDF curves. Using the same methodology used in Equation I, the parameters of Equation II were obtained.

To evaluate the performance and quality of the fit of the two equations (I and II), the following statistical measures were used: Mean squared error for the standard deviation of measured data, which standardizes the mean squared error using the standard deviation of the observations to assess the performance of the fitted model. The Nash-Sutcliffe coefficient, which is a normalized statistic that determines the relative magnitude of the residual variance compared to the variance of the observed data, was used to assess the efficiency of the model. The Percentage Trend Index was used to identify and assess the accuracy of the model's tendency to underestimate or overestimate. Finally, the coefficient of determination (R²), obtained from the relationship between estimated and observed precipitation, was used to assess the degree of adherence between these variables.

RESULTS

The results of the randomness and temporality tests indicate the absence of human interference and the effect of time on the behavior of the rainfall dataset (Table 1). The values of the number of inflections observed in the series, number of sample data, variance, significance level, equation coefficient, and variance coefficient were suitable for this type of adjustment. The homogeneity of the rainfall dataset was confirmed using the double-mass test (Figure 1). The relationship between the data showed that there was no change in the slope of the model's fit line in relation to the points. The degree of adherence to the data resulted in a very high correlation coefficient (R^2 = 0.999) between the data.

The absolute values obtained with the Anderson-Darling adhesion test, through which the adjustment of the Gumbel statistical distribution to the observed data was verified, allowed us to conclude that the distribution was adjusted to the data (Table 1). The accuracy of the adjustment of the probabilistic distribution, through the concordance index (d), showed perfect concordance between the predicted values and the observed values because results close to one (1.0) mean concordance and close to zero (0) total discordance. Precision was measured by the correlation coefficient $(R = 0.997)$, which indicated a minimum degree of dispersion of the data obtained in relation to the average, that is, high precision according to the classification proposed by HOPKINS (2000).

Table 1. Results of nonparametric tests applied to the data set of the historical series of rainfall intensities, the Willmott precision or agreement index (d), the correlation coefficient (R) and the performance index (C), obtained in the calculation of the Gumbel distribution, for the city of Lages over 28 years (1989 - 2016)

Ni: number of inflections observed in the series; N: number of sample data; E (Ni): expectation for Ni; Var (Ni): variance of Ni; α: significance level; | t |: statistical test; Cs: coefficient of the equation; Var (cs): coefficient of variance "cs".

Figure 1. Curve adjusting the precipitation values of the city of Lages to the precipitation values of EPAGRI Meteorological Station using the double mass analysis method.

Using the Gumbel distribution parameters, it was possible to calculate the maximum rainfall intensity values and compare them with the observed maximum rainfall intensity data (Figure 2). The almost perfect fit was evident in the IDF relationship, which resulted in a straight-line relationship between the values (R $2 =$ 0.999), that is, with a slope of the straight line close to one. This indicates how well the probability distribution fits the data.

Figure 2. Relationship between observed and calculated rainfall intensity values for durations of up to 240 min in the city of Lages.

The average rainfall intensity decreased exponentially with increasing duration (Figure 3), regardless of the period and return (Tr) of rainfall, in accordance with the classical behavior normally observed in this relationship. Naturally, the average value of rainfall intensity increased with the increase in the Tr value of precipitation.

Figure 3. Intensity-duration-frequency (IED) curves resulting from the relationship between the maximum average precipitation and the duration of these rains of up to 240 minutes (Equation I), obtained with rains between 1989 and 2016, for the return periods of 2, 5, 10, 25, and 50 years, in the city of Lages.

The relationship between the calculated and observed rainfall intensities resulted in an excellent fit of the model to the data, with a coefficient of determination $(R²)$ equal to 0.990 (Figure 4). Furthermore, the slope of the straight line was 44.06° for the angular coefficient of the straight line (0.968). This resulted in the excellent adherence of Gumbel's theoretical data series to the observed precipitation values.

Figure 4. Relationship between observed rainfall intensity and calculated rainfall intensity for durations of 360 to 1,440 min in the city of Lages.

The fitted parameters and the intensity-duration-frequency (IDF) II equation for the data recorded at the meteorological station under study are presented in Table 2. Equation II, adjusted from the disaggregation of the maximum precipitation of 1 (one) day, represents the IDF equation for the study site, which allows the calculation of the maximum rainfall intensity for times lasting from 360 to 1,440 minutes and the return of up to 50 years.

The average rainfall intensity decreased exponentially with increasing duration (Figure 5), regardless of the return period (Tr) of the rainfall, in accordance with the classical behavior normally observed in this relationship. Naturally, the average value of rainfall intensity increased with the increase in the Tr value of precipitation.

Figure 5. Intensity-duration-frequency (IDF) curves resulting from the relationship between the maximum average precipitation and the duration of these rains between 360 minutes and 1440 minutes (Equation II), obtained with rains between 1989 and 2016, for return periods of 2, 5, 10, 25, and 50 years, in the city of Lages.

The adjusted equation II shown in Table 2 can be used to predict heavy rainfall lasting up to 360 and 1,440 min in the city of Lages. The heavy precipitation predicted by this equation is valid for return periods of

less than 50 years. The return period should be expressed in units of years and the duration of rainfall in units of minutes.

The final phase of obtaining the adjusted equation (I) consisted of adjusting the equation to the index and parameters that resulted in the precipitation values in Table 2. For practical reasons and to minimize errors, two truncated decimals were adopted in the value of the performance index "C" and in the data adjustment parameter "t 0", and three truncated decimals were adopted in the value of the performance index "m" and in the adjustment parameter "n". The adjusted equation shown in Table 2 can be used to predict heavy rainfall lasting up to 240 min in Lages. The adjusted equation II shown in Table 2 can be used to predict heavy rainfall lasting up to 360 and 1,440 min in the city of Lages. In equations I and II, the predicted heavy precipitation is valid for return periods of less than 50 years, and the return period must be expressed in unit years and the duration of rainfall in unit minutes.

Table 2. Adjusted parameters and intensity-duration-frequency (IDF) equation for Equations I and II to be used in forecasting rainfall intensity for duration times (td) less than 240 minutes and (td) from 360 to 1,440 minutes, respectively, for the city of Lages

Parameter				IDF Equation I:
	m	ιρ	n	
1,285.65	0.133	14.15	0.865	1,285.65 $Tr^{0.133}$ $\frac{1}{(t_d+14.15)^{0.865}}$
Parameter				IDF II Equation
С	m		n	
1.923.06	0.188	34.85	0.86	1,923.06 $Tr^{0.188}$ $i (mm h^{-1})$. $-\frac{t_d + 34.85)^{0.86}}{t_d + 34.85)^{0.86}}$

C: performance index; m, t0 and n: tuning parameters.

The first test applied was the Root Mean Square Error (RMS). In this test, the result can range from zero, indicating a perfect simulation model, to a high positive value, indicating an imperfect simulation model. The values obtained from the RMS test were 0.041 and 0.040 for Equations I and 0.040 for Equation II (Table 3). These values indicate residual variation close to zero. The lower the residual value, the lower the mean squared error, and the better the model performance, according to MORIASI et al. (2007).

Then, the test known as Nash-Sutcliffe (NS) coefficient was applied to verify the efficiency of the IDF equation model (NASH & SUTCLIFFE 1970). The results for both equations were 0.998, which indicates excellent performance (Table 3). The range of acceptable values for a given test ranges from 0 to 1. In the case of a negative value, the mean of the observed data may provide better prediction than the values predicted by the model.

The third test applied, the Percentile Bias Index (PBIAS), which recommends almost zero results for an adequate simulation of the model, resulted in values with similar magnitudes and signs. For Equation I, the value was 0.476, and for Equation II, it was 0.338 (Table 3). Both equations provided excellent results because low values indicate good accuracy in the model simulation. Furthermore, according to MORIASI et al. (2007), negative values indicate a tendency to overestimate the model. Still, in the last test, similar results were obtained for the two equations, with values of 0.979 and 0.970 for Equations I and II, respectively. The values of the coefficient of determination (R ²) can vary from 0 (zero) to 1. Here, zero indicates no correlation and 1 indicates perfect correlation. The result of the two equations approached the unitary value, indicating how much the variance of the observed values was replicated by predicting the adjusted model.

DISCUSSION

The randomness test results indicated that the oscillations in the rainfall series were due to natural causes without anthropogenic interference. Regarding the hypothesis of stationarity, the hypothesis that the dataset did not present temporal trends was confirmed; that is, there were no clear cycles throughout the observation period. In general, historical precipitation data are required to apply probability distributions (Table 1). The adherence test applied to the data confirmed the high reliability of the probabilistic distribution and can be adopted for heavy rainfall data from the study site. Similar results were reported by BACK (2006), COLOMBELLI & MENDES (2013), and CARDOSO et al. (2014). These authors obtained satisfactory values for adherence to the Gumbel probabilistic distribution of the data, using a significance level of 0.1, in the cities of Videira, Chapecó, and Lages, respectively, in the state of Santa Catarina.

The criterion proposed by CAMARGO & SENTELHAS (1997) was adopted to interpret the degree of adjustment of the Gumbel distribution using the **C** index (Figure 1). Based on the results presented in Table 1 and the classification of the resulting values according to WILLMOTT (1981), CAMARGO & SENTELHAS (1997), and HOPKINS (2000), we conclude that the Gumbel distribution was perfectly acceptable. This acceptance is possible because the combination of the test results revealed a high degree of approximation between the estimated and observed values.

According to BERTONI and TUCCI (2001), when the average rainfall intensity was related to duration, the values varied inversely for the same return time. This typical behavior was identified in all curves of Equation I when relating the intensity to the duration of the rain (Figure 2). Analyzing the IDF curves estimated by the Gumbel distribution from the disaggregation method (Equation II), it can be observed that as the duration of the rain increased, its intensity decreased, as identified and described previously, typical of IDF relationships.

Using data from the annual maximum precipitation series of the Meteorological Station in the city of Lages between 2000 and 2009, CARDOSO et al. (2014) obtained the IDF equation from the analysis of daily diagrams, presented by the following notation: I=2.050tr0.32 (t+42.3) -1.21. In this equation, **i** is the maximum average intensity, Tr is the return time, and t is the duration of the rain. CARDOSO et al. (2014) established a single equation to be used for duration times of 1 h to 24 h. In this study, to obtain greater precision, two equations were adjusted for different rainfall duration times (Equation I and II).

After adjusting the parameters of two equations (I and II), some tests were applied to verify the quality of the adjustments (Table 3). From the results, based on the four tests used, it was then considered that the two equation models (I and II) were calibrated satisfactorily, as the calculated values of the objective function presented values higher than the minimum quality requirements for both equations. Thus, these equations can be used to predict rainfall in Lages.

CONCLUSION

The frequency distribution of rainfall intensity in the city of Lages, estimated by the Gumbel method using the Kolmogorov-Smirnov test (p <0.05), was adjusted to the frequency of observed rainfall. This confirms the reliability of the probabilistic distribution of the data and can be used to estimate the intensity for given return times and rainfall durations. The intensity-duration-frequency (IDF) equation model fits observed precipitation data over a 28-year series in the city of Lages and can be applied to predict the maximum mean rainfall intensity. The IDF I equations are written as $I=1,285.65$ tr 0.133 (t+14.2) -0.87 and are applied to rain periods of up to 240 min. The IDF II equations are written as I=1,500.42tr 0.189 (t+42.1) -0.82 and are applied for periods of rain duration between 360 and 1,440 minutes.

ACKNOWLEDGMENT

To the National Council for Scientific and Technological Development (CNPq) for the PQ scholarship of the second author and for the financial resources to develop the research; and to FAPESC for part of the financial resources spent on the research.

REFERENCES

BACK AJ. 2006. Relação intensidade-duração-frequência de chuvas intensas de Chapecó, Estado de Santa Catarina. Acta Scientiarum Agronomy 28: 575-581.

BERTONI JC & TUCCI CEM. 2001. Precipitação. In: TUCCI CEM. (Org.). Hidrologia: ciência e aplicação. 2.ed. Porto Alegre: ABRH. p.177-235.

- CAMARGO AP & SENTELHAS PC. 1997. Avaliação do desempenho de diferentes métodos de estimativa da evapotranspiração potencial no Estado de São Paulo. Revista Brasileira de Agrometeorologia 5: 89-97.
- CARDOSO CO et al. 2014. Generation of intensity duration frequency curves and intensity temporal variability pattern of intense rainfall for Lages/SC. Brazilian archives of biology and technology 57: 274-283.

CETESB. 1979. Drenagem urbana: Manual de projeto. São Paulo: Publisher. 479p.

CHOW VT. 1959. Open-Chanel hydraulics. New York: Musgraw-Hill.

- COLOMBELLI C & MENDES R. 2013. Determinação dos parâmetros da equação de chuvas intensas para o município de Videira, SC. UNOESC & Ciência 4: 169-180.
- DAEE-CETESB. 1986. Departamento de Águas e Energia Elétrica do Estado de São Paulo Companhia de Tecnologia de Saneamento Ambiental. Drenagem urbana – Manual de projeto. 3.ed. São Paulo: CETESB. 464p.
- HOPKINS W G. 2000. Correlation coefficient: a new view of statistics. Available in: <http://www.sportsci.org/resource/stats/correl.html>. Accessed on: 10 Jan 2017.
- MELLO CR & SILVA A M. 2013. Hidrologia: princípios e aplicações em sistemas agrícolas. Lavras: UFLA. 455p.
- MORIASI DN et al. 2007. Model Evaluation Guidelines for Systematic Quantification of Accuracy in Watershed Simulations. Transactions of the ASABE 50: 885-900.
- NASH JE & SUTCLIFFE JE. 1970. River flow forecasting through conceptual models: Part I. A discussion of principles. Journal of Hydrology 10: 282-290.
- OLIVEIRA LFC et al. 2005. Intensidade-duração-frequência de chuvas intensas para localidades no Estado de Goiás e Distrito Federal. Pesquisa Agropecuaria Tropical 35: 13-18.
- SILVA DD et al. 1999. Estimativa e espacialização dos parâmetros da equação de intensidade-duraçãofrequência da precipitação para o Estado de São Paulo. Revista Engenharia na Agricultura 7: 70-87.

TUCCI CEM (Org). 1993. Hidrologia: ciência e aplicação. Porto Alegre: UFRGS. 943p.

VILLELA SM & MATTOS A. 1975. Hidrologia Aplicada. São Paulo: Mcgraw-Hill do Brasil. 245p.

WILKEN PS. 1978. Engenharia de Drenagem Superficial. São Paulo: CETESB. 477p.

WILLMOTT CJ. 1981. On the validation of models. Physical Geography 2: 184-194.